The Paradoxes of the Interaction-free Measurements

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Interaction-free measurements introduced by Elitzur and Vaidman [Found. Phys. **23**, 987 (1993)] allow finding infinitely fragile objects without destroying them. Paradoxical features of these and related measurements are discussed. The resolution of the paradoxes in the framework of the Many-Worlds Interpretation is proposed.

Key words: Interaction-free Measurements; Quantum Paradoxes.

I. Introduction

The interaction-free measurements proposed by Elitzur and Vaidman [1, 2] (EV IFM) led to numerous investigations and several experiments have been performed [3 - 17]. Interaction-free measurements are very paradoxical. Usually it is claimed that quantum measurements, in contrast to classical measurements, invariably cause a disturbance of the system. The IFM is an example of the opposite: this is a quantum measurement which does not lead to any disturbance, while its classical counterpart invariably does.

There are many ways to understand the interaction-free nature of the EV IFM. A detailed analysis of various interpretations appears in [18, 19]. In this paper I will concentrate on the paradoxical aspects of interaction-free measurements. In Sect. II I describe the IFM of Renninger [20] and Dicke [21]: changing the quantum state of a system without interaction. In Sect. III the original proposal of Elitzur and Vaidman is presented and the basic paradox of the EV IFM is discussed: a particular interaction leads to an explosion, nevertheless, it can be used for obtaining information without the explosion. Section IV is devoted to another paradoxical feature of the EV IFM: obtaining information about a region in space without anything coming in, out, or through this place. It also includes a brief analysis of the "delayed choice experiment" proposed by Wheeler [22] which helps to define the context in which the above claims, that the measurements are interaction-free, are legitimate. Section V is devoted to the variation of the EV IFM proposed by Penrose [23] which, instead of testing for the presence of an object in a particular place, tests a certain property of the object in an interaction-free way. Section VI introduces the EV IFM procedure for a quantum object being in a superposition of different locations. It works equally well: it collapses the spatial quantum state of the object to a particular place without any disturbance of its internal state. However, the second paradoxical feature of the EV IFM, i. e. the fact that nothing has been in the vicinity of the object the presence of which was discovered, has a subtle constraint. This point is explained in Sect. VII via the analysis of Hardy's paradox [24]. I conclude the paper in Sect. VIII by arguing that the paradoxes of IFM disappear in the framework of the many-worlds interpretation.

I want to mention a naive paradox which I have heard several times and which I do not discuss in this paper (I discussed it in [18]). Finding the position of a particle in an interaction-free way means, in particular, (according to these arguments) finding it without changing its momentum. Thus, a high precision experiment of this kind performed on a particle with bounded momentum uncertainty leads to

breaking the Heisenberg uncertainty relation. This type of arguments appear to be due to the misleading identification of the EV IFM with an experiment without momentum (energy) transfer [25 - 27].

II. The IFM of Renninger and Dicke: Negative Results Experiment

The paradox of the Renninger-Dicke type measurement is that it causes some changes in the state of the system "without interaction." Renninger discussed a *negative result experiment*: a situation in which the detector does not detect anything. In spite of the fact that nothing happened to the detector, there is a change in the measured system. He considered a spherical wave of a photon after it extended beyond the radius at which a scintillation detector was located in part of the solid angle, see Figure 1. The state of the detector remained unchanged but, nevertheless, the wave-function of the photon is modified. The name "interaction-free" for Renninger's setup might be justified because there is not *any*, not even an infinitesimally small, change in the state of the detector in the described process. This is contrary to the classical physics in which interaction in a measurement process can be made arbitrary small, but it cannot be exactly zero.

Dicke's paradox is the apparent non-conservation of energy in a Renninger-type experiment. He considered an atom in a ground state inside a potential well. Part of the well was illuminated by a beam of photons. A negative result experiment was considered in which no scattered photons were observed, see Figure 2. The atom changed its state from the ground state to some superposition of energy eigenstates (with a larger expectation value of the energy) in which the atom does not occupy the part of the well illuminated by the photons, while photons (the measuring device) apparently have not changed their state at all, and he asked: "What is the source of the additional energy of the atom?!"
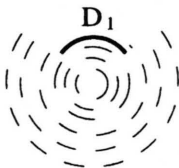

Fig. 1. **Renninger's experiment.** The photon spherical wave is modified by the scintillation detector D_1 in spite of the fact that it detects nothing.
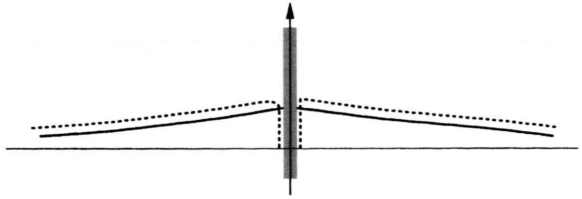

Fig. 2. **Dicke's Experiment.** The ground state of a particle in the potential well (solid line) is changed to a more energetic state (dashed line) due to short radiation pulse, while the quantum state of the photons in the pulse remains unchanged.

Careful analysis [28, 29] (in part, made by Dicke himself) shows that there is no real paradox with conservation of energy, although there are many interesting aspects in the process of an ideal measurement [30]. One of the key arguments is that the photon pulse has to be well localized in time and, therefore, it must have a large uncertainty in energy.

In the Renninger argument, the paradox exists only in the formalism of quantum mechanics and, moreover, only within some interpretations of quantum theory. The "change" which occurred "without interaction" is the change of a quantum state. Adopting the interpretation according to which the quantum state does not have its own "reality", but is a description of our knowledge about the object, removes the paradox completely: of course, negative result experiments provide us with some information which, consequently, changes our knowledge about the object, i.e., the object's quantum state.

The IFM of Elitzur and Vaidman, which will be discussed in the next section, is not concerned with changing the object without interaction, but with obtaining information about the object without interaction. The negative results experiment of Renninger and Dicke also provide some information without interaction. We learn, without interaction with the object, where the object *is not*. This is not too surprising: if the object is not in the vicinity of the detector, then the detector does not interact with it. We even can get information where the object *is* in this manner provided we have *prior information* about the state of the object. If it is known in advance that the object is somewhere inside two places and it was not found in one, obviously, we then know that it is in the second place. But this has a trivial classical counterpart: If it is known in advance that the object is in one of two separate boxes and we open one and do not see it

there, then, obviously, we know that it is in the second box, and we have not interacted with the object.

III. The Elitzur-Vaidman Interaction-free Measurements

In the EV IFM paper the following question has been considered:

> Suppose there is an object such that *any* interaction with it leads to an explosion. Can we locate the object without exploding it?

The EV method is based on the Mach-Zehnder interferometer. A photon (from a source of single photons) reaches the first beam splitter which has a transmission coefficient $\frac{1}{2}$. The transmitted and reflected parts of the photon wave are then reflected by the mirrors and finally reunite at another, similar beam splitter, see Fig. 3a. Two detectors are positioned to detect the photon after it passes through the second beam splitter. The positions of the beam splitters and the mirrors are arranged in such a way that (because of destructive interference) the photon is never detected by one of the detectors, say D_2, and is always detected by D_1.
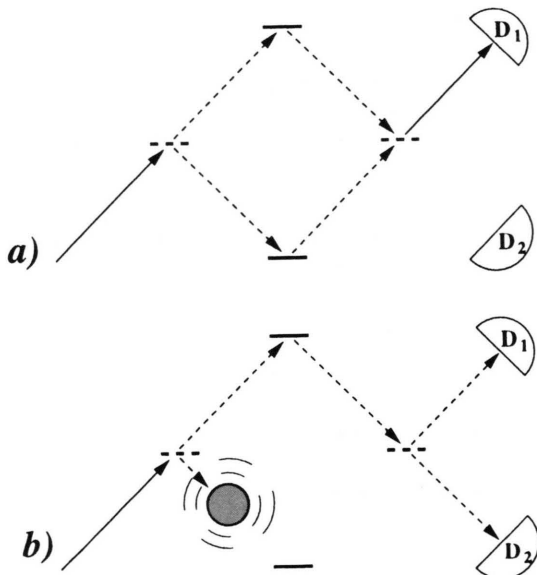

Fig. 3. (a) When the interferometer is properly tuned, all photons are detected by D_1 and none reaches D_2. (b) If the bomb is present, detector D_2 has the probability 25% to detect the photon sent through the interferometer, and in this case we know that the bomb is inside the interferometer without exploding it.

This interferometer is placed in such a way that one of the routes of the photon passes through the place where the object (an ultra-sensitive bomb) might be present (Fig. 3b). A single photon passes through the system. There are three possible outcomes of this measurement: i) explosion, ii) detector D_1 clicks, iii) detector D_2 clicks. If detector D_2 clicks (the probability for that is $\frac{1}{4}$), the goal is achieved: we know that the object is inside the interferometer and it did not explode.

The EV method solves the problem which was stated above. It allows finding with certainty an infinitely sensitive bomb without exploding it. The bomb might explode in the process, but there is at least probability of 25% to find the bomb without the explosion. "Certainty" means that when the process is successful (D_2 clicks), we know for sure that there is something inside the interferometer. (A modification of the EV IFM which employs the quantum Zeno effect allows to reduce the probability of the explosion to an arbitrarily small value [3].)

In his earlier paper, Renninger [31] considered an experimental setup which is almost identical to that of the EV IFM: a Mach-Zehnder interferometer tuned to have a dark output towards one of the detectors. However, he never regarded his experiment as a measurement on an object which was inside the interferometer: Renninger's argument, as in the experiment described in Fig. 1, was about "interaction-free" changing the state of the photon. Renninger has not asked the key question of the EV IFM: How to get information in an interaction-free manner.

The basic paradox of the EV IFM can be presented in the following way. The only interaction of the bomb with an external world is through its explosion. Nevertheless, the EV scheme allows finding the object without the explosion. It is different from the trivial way of the Renninger-Dicke IFM when we know before the experiment that the bomb is somewhere in a particular region and then, not finding it in part of the region, tells us that it is in the remaining part. The EV method works even if we do not have *any* information about the location or even existence of the object prior to the measurement.

The weakness of this paradox can be seen in sentences: "Suppose there is an object such that *any* interaction with it leads to an explosion," "The only interaction of the bomb with an external world is through its explosion." Quantum mechanics precludes the existence of such objects. Indeed, a good model for an

"explosion" is an inelastic scattering [32]. The Optical Theorem [33] tells us that there cannot be an inelastic scattering without some elastic scattering. Thus, if it were such an object, the quantum experiment of the EV IFM would find it without interaction, but since quantum theory ensures that there are no objects like this, it also avoids the paradox. Not exactly. The EV method is still very paradoxical: it employs the explosion for detection but it does not cause the explosion (at least in some cases.) The task of the IFM can be rephrased in the following way:

> Suppose there is an object such that a particular interaction with it leads to an explosion. Can we locate the object without exploding it using this interaction?

IV. Second Paradox of the EV IFM: Measurement "without Touching"

Suppose there is a place in the Universe that no particle, no light, nothing whatsoever visited, i. e. no particle passed through this place, no particle went to this place and was stopped there. Suppose also that nothing came out of this place: no particle, no field, no source of potential observable through the Aharonov-Bohm type effect, nothing whatsoever. It seems that in this case we cannot know: Is there something in this place? If, however, we put the mirrors of the EV interferometer around this place such that one of the arms of the interferometer crosses it and send through the interferometer a single photon which ends up in D_2, then we know that there is something there. Moreover, if we later find out that this "something" is a nontransparent object then we can claim that we have found it without "touching": nothing was in the vicinity of the object.

This claim, again, has to be taken in an appropriate context. It has the same justification as Wheeler's delayed choice experiment analysis [22]. One of the "choices" of Wheeler's delayed-choice experiment is an experiment with a Mach-Zehnder interferometer in which the second beam splitter is missing, see Figure 4. In a run of the experiment with a single photon detected by D_2, it is usually accepted that the photon had a well defined trajectory: the upper arm of the interferometer. In contrast, according to the von Neumann approach, the photon was in a superposition inside the interferometer until the time when one part of the superposition reached detector D_2 (or until the
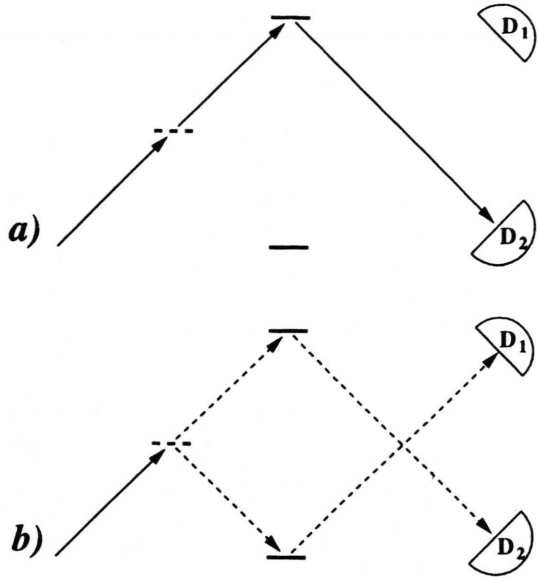

Fig. 4. (a) The "trajectory" of the photon in the Wheeler experiment given that D_2 detected the photon as it is usually described. The photon cannot leave any physical trace outside its "trajectory". (b) The "trajectory" of the quantum wave of the photon in the Wheeler experiment according to the von Neumann approach. The photon remains in a superposition until the collapse which takes place when one of the wave packets reaches a detector.

time the other part reached detector D_1 if that event was earlier). At that moment the wave function of the photon collapses to the vicinity of D_2.

The justification of Wheeler's claim that the photon detected by D_2 never was in the lower arm of the interferometer is that, according to the quantum mechanical laws, we cannot see any physical trace from the photon in the lower arm of the interferometer. This is true if (as it happened to be in this experiment) the photon from the lower arm of the interferometer cannot reach the detector D_2. The fact that there cannot be a physical trace of the photon in the lower arm of the interferometer can be explained in the framework of the two-state vector formulation of quantum mechanics [34, 35]. This formalism is particularly suitable for this case because we have a pre- and post-selected situation: the photon was post-selected at D_2. Thus, while the wave function of the photon evolving forward in time does not vanish in the lower arm of the interferometer, the backward-evolving wave function does. Vanishing of one of the waves (forward or backward) is enough to ensure that the photon cannot

cause any change in local variables of the lower arm of the interferometer.

In the EV IFM we have the same situation. If there is an object in the lower arm of the interferometer, see Fig. 3b, the photon cannot go through this arm to detector D_1. This is correct if the object is such that it explodes whenever the photon reaches its location and we have not observed the explosion. Moreover, this is also correct in the case in which the object is completely intransparent and it blocks the photon in the lower arm eliminating any possibility of reaching D_1. Even in this case, when the object does not explode on touching, we can claim that we locate the object "without touching". This claim is identical to the argument according to which the photon in Wheeler's experiment went solely through the upper arm.

In the framework of the two-state vector approach this is explained in the following way. The forward-evolving quantum state vanishes in the lower arm of the interferometer beyond the location of the object, while the backward-evolving wave function vanishes before the location of the object. Thus, at every point of the lower arm of the interferometer one of the quantum states vanishes. This ensures that the photon cannot make any physical trace there. Note, that the two-state vector formalism itself does not suggest that the photon is not present at the lower arm of the interferometer; it only helps to establish that the photon does not leave a trace there. The latter is the basis for the claim that in some sense the photon was not there.

V. The Penrose Interaction-free Measurements

The task of the EV IFM is to find the location of an object without interaction. Penrose proposed to use a similar idea for testing some property of an object without interaction [23]. The object is again a bomb which explodes when anything, even a single photon, "touches" its trigger device. Some of the bombs are broken (they are duds) in a particular way: their trigger device is locked to a body of the bomb and no explosion and no motion of the trigger device would happen when it is "touched". Again, the paradox is that any touching of a good bomb leads to an explosion. How can we test the bomb without exploding it?

In the Penrose version of IFM, the bomb plays the role of one mirror of the interferometer, see Figure 5. It has to be placed in the correct position. We are allowed to do so by holding the body of the bomb. However, the uncertainty principle puts limits on placing the
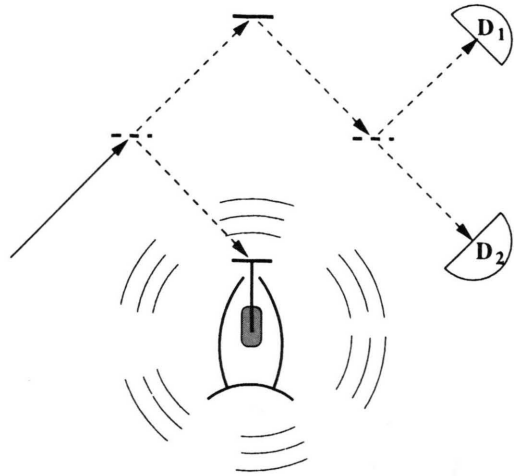

Fig. 5. **The Penrose bomb-testing device.** The mirror of the good bomb cannot reflect the photon, since the incoming photon causes an explosion. Therefore, D_2 sometimes clicks. (The mirror of a dud is connected to the massive body, and therefore the interferometer "works", i.e. D_2 never clicks when the mirror is a dud.)

bomb in its place before the experiment [36]. Only if the position of the bomb (in fact, what matters is the position of a dud) is known exactly, the limitations are not present. In contrast, in the EV IFM the bomb need not be localized prior to the measurement: the IFM localizes it by itself.

VI. Interaction-free Localization of a Quantum Object

When the EV IFM is applied to a quantum object spread out in space, it collapses the spatial wave function without changing the state of internal variables [2]. Let us discuss two aspects of such experiments.

First, in order to see the difference between the Renninger-Dicke IFM and the EV IFM more vividly, let us consider an application of the EV method to Dicke's experimental setup. Instead of the light pulse we send a "half photon": We arrange the EV device such that one arm of the Mach-Zehnder interferometer passes through the location of the particle, see Figure 6. Then, if detector D_2 clicks, the particle is localized in the interaction region.

In both cases (the Renninger-Dicke IFM and this EV IFM) there is a change in the quantum state of the particle without apparent scattering of the photon by the particle. However, the situations are quite different. In the original Dicke's experiment we can
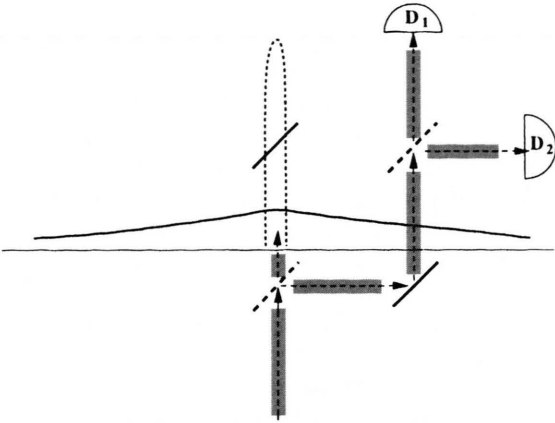
Fig. 6. **The EV modification of Dicke's Experiment.** The ground state of a particle in the potential well (solid line) is changed to a well localized state (dashed line) when the photon is detected by the detector D_2.

claim that the dashed line of Fig. 2 is the state of the particle after the experiment only if we have prior information about the state of the particle before the experiment (solid line of Fig. 2). In contrast, in the EV modification of the experiment, we can claim that a particle is localized in the vicinity of the interaction area (dashed line of Fig. 6) even if we had no prior information about the state of the particle.

The second aspect of the EV IFM applied to quantum objects is that the argument according to which the measurement was performed without a photon being in the vicinity of the object, encounters a subtle difficulty: it might be the case that we perform the procedure of the IFM, obtain the photon click at D_2, but, nevertheless, the photon was *with certainty* in the area of interaction.

First, let us repeat the argument which led us to think that the photon was not there. If D_2 clicks, we can argue that the particle had to be on the way of the photon in the left arm of the interferometer (in the right arm the trajectories do not intersect), otherwise, it seems that we cannot explain the arrival of the photon to the "dark" detector D_2. If the particle was on the way of the photon in the left arm of the interferometer we can argue that the photon was not there, otherwise we had to see the explosion. Therefore, the photon went through the right arm of the interferometer and it was not present in the left arm of the interferometer.

The persuasive argument of the previous paragraph is wrong! Not just the semantic point discussed above,

i. e., that the quantum wave of the photon in the left arm of the interferometer in the part before the "meeting point" with the particle was not zero, is incorrect. It is wrong to say that the photon was not in the left arm even in the part *beyond* the meeting point with the atom. In an unambiguous operational sense it is wrong to say that in the experiment in which D_2 clicks, the probability to find (in a non-demolition way) the photon in the left arm of the interferometer after the meeting point with the atom is zero. The photon *can* be found in the left arm of the interferometer! A particular way to achieve this is discussed in the next section.

VII. Hardy's Paradox

Hardy considered "nested interaction-free measurements" [24]. The particle is in a superposition of two wave packet inside its own Mach-Zehnder interferometer (see Fig. 7.) If D_2 (for the photon) clicks, the particle is localized inside the interferometer. If we assume that before the experiment the whole volume of the interferometer except the "meeting place" W which we want to test was found empty, we can claim that the click of D_2 localizes the particle inside W.
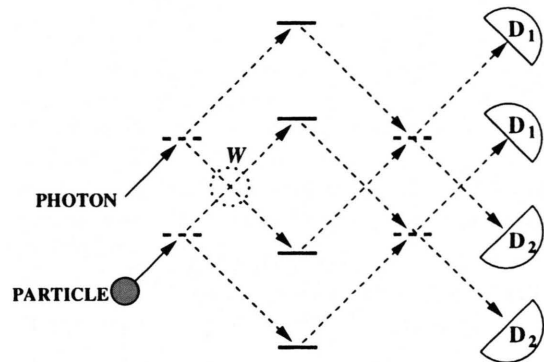

Fig. 7. **Hardy's Paradox.** Two interferometers are tuned in such a way that, if they operate separately, there is a complete destructive interference towards detectors D_2. The lower arm of the photon interferometer intersects the upper arm of the particle interferometer in W such that the particle and the photon cannot cross each other. When the photon and the particle are sent together (they reach W at the same time) then there is a nonzero probability for clicks of both detectors D_2. In this case one can infer that the particle was localized at W and also that the photon was localized at W. However, the photon and the particle were not present in W together. This apparently paradoxical situation does not lead to a real contradiction because all these claims are valid only if tested separately.

However, the particle plays the role of the photon of another IFM (we can consider a gedanken situation in which the particle which explodes when the photon reaches its location can, nevertheless, be manipulated by other means). If this other IFM is successful (i. e. "D_2" for the particle clicks) then the other observer can claim that she localized the photon of the first experiment in the "meeting place" W, i. e. that the photon passed through the lower arm of the interferometer on its way to D_2.

Paradoxically, both claims are true (in the operational sense): the first experiment localizes the particle in W, and the second, at the same time, localizes the single photon there. Both claims are true separately, but not together: if we would try to find both the photon and the particle in W, we will fail with certainty. Such peculiarities take place because we consider a pre- and post-selected situation (the post-selection is that in both experiments detectors D_2 click) [37].

In spite of this peculiar feature, the experiment is still interaction-free in the following sense. If somebody would test the success of our experiment for localization of the particle, i. e. would measure the particle's location shortly after the "meeting time" between the particle and the photon, then we know for sure that she would find the particle in the left arm of the interferometer and, therefore, the photon cannot be there. Discussing the issue of the presence of the particle with her, we can correctly claim that in our experiment the photon was not in the vicinity of the particle. Again, at the end of the EV IFM procedure for a quantum object we cannot claim that the photon we used was not in the vicinity of W. But we can still claim that we have localized the particle there without the photon being in W. To localize means that if tested it must be found there, and if tested, the photon was not there. However, if, instead of measuring the position of the particle after the meeting time, she finds the particle in a particular superposition, she can claim with certainty that the photon was in W. (Compare this with *deterministic quantum interference experiments* [38]).

VIII. Resolution of the Paradoxes in the Framework of the Many-Worlds Interpretation

I want to argue that the paradoxes presented above are resolved, or at least appear less paradoxical in the framework of the Many-Worlds Interpretation

(MWI) [39]. The MWI itself has several interpretations, some of which are conceptually different. I take the view in which we have *one* physical universe which incorporates many (subjective) worlds [40]. The physical universe is described by one wave function evolving deterministically according to the Schrödinger equation. This wave function can be decomposed into a superposition of many states, each corresponding to a different story. One of the stories is the world as *you*, the reader of this paper, know it. What we perceive is just a small part of what *is* in the universe. The laws of physics relate to the whole universe, and it is not surprising that considering only a part of it leads to paradoxical situations. Considering the physical universe, i. e. all worlds together, resolves the paradoxes [41].

In the framework of the MWI it is not true that we get information about the region without anything being there. It is not true that we find the bomb without an explosion. The photon which we sent into the interferometer was there, but – in another world. In our experiment three worlds (three different stories) appear:

(i) there is an explosion,

(ii) detector D_1 clicks,

(iii) detector D_2 clicks.

Obtaining information in the world (iii) without any object being in the region became possible because in the world (i) a photon was in the region and it caused the explosion.

The EV IFM allows to find an object in a particular place without visiting the place. However, it does not allow to find out that the place is empty without being there. The MWI explains why it is so. If the place is empty, then in the EV procedure there is only one world, the one we are aware of, so obtaining information about the region without being there is on the level of the whole universe. Our physical intuition correctly tells us that such a situation is impossible.

What I can see in common between the Renninger-Dicke IFM and the EV IFM is that in the framework of the many-worlds interpretation in both cases we can see the "interaction": radiation of the scintillator in the Renninger experiment or explosion of the bomb in the EV experiment, but these interactions take place in the "other" branch, not in the branch we end up discussing the experiment. (In an attempt to avoid adopting the many-worlds interpretation such

interactions were considered as *counterfactual*; see [23] p. 240 and [42]).

To conclude this paper let me express my attitude to quantum paradoxes. Paradoxes in physics are very important: they lead to new theories. There are numerous paradoxes in quantum mechanics, but, in my view, none of them is a real paradox which will lead to new physical laws. The quantum mechanical paradoxes do not follow from incorrectness or incompleteness of quantum theory, but from inappropriate classical intuition which people developed during thousands of years when quantum phenomena were not observed and thus no one had reason to believe in quantum mechanics. The role of quantum paradoxes is not to lead to new theories but to lead to the development of new intuition about our world. Here, I probably will be in a minority: I prefer to believe that there is no conceptually new physics which we do not know yet, I prefer the feeling that we, basically, do understand our world.

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